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Probing Maldacena-Nunez in IR with $p \overline{D3}$ branes

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Abstract

We probed Maldacena-Nunez solution in IR with p coincident anti D3 branes and found that these probe branes become a fuzzy NS5 brane. Doing the dual analysis i.e. from the NS5 brane point of view with the charge of p anti D3 brane on the world-volume of NS5 brane, we showed that to leading order this potential matches with that of p anti D3 branes and the potential on the NS5 brane has a stable minima and have also calculated the potential, from the NS5 brane point of view, for a small fluctuation along the radial direction.

1 Introduction

In recent years there are several interesting attempts to understand the holographic dual of various supergravity background for which the Yang-Mills theory will be less supersymmetric. The examples are Klebanov-Strassler background [1] and Maldacena-Nunez backgrounds [2]. In the former case, KS have considered a stack of N D3 branes along with M fractional D3 branes, which are D5 branes wrapped over a collapsing two cycle, in deformed conifold in Type IIB string theory. This supergravity background is smooth¹. The corresponding Yang-Mills theory is $\mathcal{N} = 1$ supersymmetric with gauge group $SU(N + M) \times SU(N)$ and possesses many interesting properties like: confinement in IR, chiral symmetry breaking and duality cascades in UV. While, in the latter case [2], MN considers N coincident NS5 branes, in Type IIB string theory, wrapped on a two sphere, S^2 and twisting the normal bundle in a precise way² such that the corresponding Yang-Mills theory will be $\mathcal{N} = 1$ supersymmetric. MN solution is also smooth, shows confinement and breaks $U(1)_R$ symmetry in UV. It's been constructed by taking the solution constructed in [4] in seven dimensional gauged supergravity and uplifting the solution to 10 dimension. The S-dual solution namely the D5 brane wrapped on S^2 with a three form field strength, F_3 , has also been presented in [2]. The major difference between KS and MN solution is that in the latter case there are no regular branes but features only fractional branes whereas in the former, it has both regular and fractional branes.

There is a search for nonsupersymmetric and stable gravitational background for which the corresponding gauge theory will also be nonsupersymmetric. The study of Kachru et al.[7] shows the presence of a nonsupersymmetric but only classically stable configuration for arbitrarily long lived and quantum mechanically unstable vacua. In this construction, [7] have considered a stack of p anti D3 branes probing the KS geometry, in particular, the tip of the deformed conifold. Near to the tip and for $\frac{p}{M} \simeq 3\%$ there exists a classically stable, nonsupersymmetric vacua but for $\frac{p}{M} \simeq 8\%$ there exists a marginally stable minimum and for $\frac{p}{M} > 8\%$ corresponds to a stable but supersymmetric vacua, where M is the flux of the fractional D3 branes. This supersymmetric vacua comes from “brane/flux annihilation” in which the flux of H_3 jumps down by one unit and there remains $M-p$ D3 branes. The

¹Unlike the Klebanov-Tseytlin background [3] where there is a naked singularity.

²The spin connection on the two sphere has been set equal to the gauge potential, so that there will be a covariantly constant Killing spinor [5, 6].

mechanism as studied in [7] is: when the p anti D3 branes are being used to probe the KS solution in IR, then these anti branes become a fuzzy NS5 branes and these NS5 branes from the north-pole perspective corresponds to M- p D3 branes and there is a fall of H_3 flux by one unit.

In a recent study of Aharony et al.[8], have shown that a small deformation to Maldacena-Nunez solution corresponds to a stable nonsupersymmetric background. In the dual theory it corresponds to giving mass to some of the scalars. In the gravitational background the stability comes from the fact that the original theory had a mass gap and a small deformation can't change it, as argued in [8].

In this paper, we want to present the probing of Maldacena-Nunez D5 brane background by a stack of p anti D3 branes in IR following the work of [7]. The analysis essentially is done using the Born-Infeld and Chern-Simon action in the S-dual frame in the above stated background. In the S-dual frame, we shall see that these anti D3 branes will couple to a six form potential B_6 , but the background has no such potential. Hence, the dynamics of these branes are described by the DBI action. Analyzing the DBI action in this background implies that these anti branes will be accumulated at $r = 0$.

First, when p coincident anti D3 branes are put at the IR of MN background it becomes a fuzzy NS5 brane due to Myers effect and second, studying the dynamics of an NS5 brane with the charge of p anti D3 brane on its world volume in the MN background reproduces the energy seen by these p anti D3 branes to leading order in $\frac{p}{N}$, where N is the number of fractional D3 branes of MN background. From the study of the fuzzy NS5 brane point of view, we found that the potential seen by this brane is stable.

The paper is organized as follows. In section 2, we shall present the Maldacena-Nunez solution of N D5 branes wrapped on a two sphere and its IR behavior. In section 3, we shall analyze the dynamics of the p coincident anti D3 brane in the IR of MN solution and show the appearance of fuzzy 5 brane and in section 4, we shall do the dual analysis i.e. from the NS5 brane point of view and show that the potential energy for the static case matches with the potential energy found in section 3. In section 5, we shall add a small fluctuation in the radial direction and repeat the analysis of section of 4.

2 Maldacena-Nunez background and its $r \rightarrow 0$ limit

The supergravity solution of N number of D5 branes wrapped on a 2-sphere, S^2 is described by the the Maldacena-Nunez background [2]. The non trivial field content of this background are metric, dilaton, ϕ and a magnetic three form field strength, F_3 , and the form of the solution in string frame in $\alpha' = 1$ units are ³

$$ds_{10}^2 = e^\phi \left[dx^\mu dx_\mu + N(e^{2h} d\Omega_2^2 + dr^2 + \frac{1}{4}(\omega^i - A^i)^2) \right], \quad (1)$$

where $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2$. The world volume direction of the branes are along x_μ, θ, ϕ . The ω^i 's are the SU(2) left invariant one forms and satisfy $d\omega^i = -\frac{1}{2}\epsilon_{ijk}\omega^j \wedge \omega^k$ algebra, which essentially parametrises a 3 sphere, S^3 . The explicit form of these ω^i 's are

$$\begin{pmatrix} \omega^1 \\ \omega^2 \\ \omega^3 \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi \sin\theta_1 & 0 \\ -\sin\psi & \cos\psi \sin\theta_1 & 0 \\ 0 & \cos\theta_1 & 1 \end{pmatrix} \begin{pmatrix} d\theta_1 \\ d\phi_1 \\ d\psi \end{pmatrix}, \quad (2)$$

with the ranges for these angles are $0 \leq \theta_i \leq \pi$, $0 \leq \phi_i < 2\pi$, $0 \leq \psi < 4\pi$. The one forms A^i 's have the following form

$$A^1 = -a(r)d\theta, \quad A^2 = a(r)\sin\theta d\phi, \quad A^3 = -\cos\theta d\phi, \quad (3)$$

with

$$\begin{aligned} a(r) &= \frac{2r}{\sinh 2r} \\ e^{2h} &= r \coth 2r - \frac{r^2}{\sinh^2 2r} - \frac{1}{4} \\ e^{-2\phi} &= e^{-2\phi_0} \frac{2e^h}{\sinh 2r}, \end{aligned} \quad (4)$$

where ϕ_0 is defined for the value of dilaton at $r = 0$. The 3 form looks like

$$F_3 = -N \left[\frac{1}{4}(\omega^1 - A^1) \wedge (\omega^2 - A^2) \wedge (\omega^3 - A^3) - \frac{1}{4}F^a \wedge (\omega^a - A^a) \right], \quad (5)$$

where F^a is defined as

$$F^a = dA^a + \frac{1}{2}\epsilon_{abc} A^b \wedge A^c. \quad (6)$$

³We are following the notation of [2, 9, 11, 10].

Explicitly, the components of it are

$$F^1 = -\frac{da}{dr}dr \wedge d\theta, \quad F^2 = \frac{da}{dr}\sin\theta dr \wedge d\phi, \quad F^3 = (1-a^2)\sin\theta d\theta \wedge d\phi. \quad (7)$$

The two form potential associated to this three form field strength, $F_3 = dC_2$, is

$$C_2 = \frac{N}{4} \left[\psi(\sin\theta d\theta \wedge d\phi - \sin\theta_1 d\theta_1 \wedge d\phi_1) - \cos\theta \cos\theta_1 d\phi \wedge d\phi_1 - a(d\theta \wedge \omega^1 - \sin\theta d\phi \wedge \omega^2) \right]. \quad (8)$$

The six form potential that follows from the equation of motion of F_3 , in Einstein frame, i.e. $d(e^\phi \star F_3) = 0$, is

$$C_6 = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge C, \quad (9)$$

where C is [11]

$$C = -\frac{N}{8}e^{2\phi} \left[[(a^2-1)a^2e^{-2h} - 16e^{2h}]\cos\theta d\phi \wedge dr - (a^2-1)e^{-2h}\omega^3 \wedge dr + \frac{da}{dr}(\sin\theta d\phi \wedge \omega^1 + d\theta \wedge \omega^2) \right]. \quad (10)$$

Before getting into the discussion of probe dynamics, let us look at first the behavior of the solution in $r \rightarrow 0$ limit. In this limit the functions $a(r), \phi(r)$ and $h(r)$ takes the following form

$$a(r) \rightarrow 1, \quad e^{2h} \rightarrow 0, \quad e^{2\phi} \rightarrow e^{2\phi_0}. \quad (11)$$

Hence, the solution in this limit becomes

$$\begin{aligned} ds_{10}^2 &\rightarrow ds_7^2 = e^{\phi_0} \left[dx_\mu dx^\mu + \frac{N}{2} d\Omega_3^2 \right], \\ F_3 &\rightarrow -\frac{N}{4}(\omega^1 - A^1) \wedge (\omega^2 - A^2) \wedge (\omega^3 - A^3) = -\frac{N}{2}g^4 \wedge g^3 \wedge g^5, \\ e^{2\phi} &\rightarrow e^{2\phi_0} \equiv g_s^2. \end{aligned} \quad (12)$$

where $d\Omega_3^2 = \frac{1}{2}(g^5)^2 + (g^3)^2 + (g^4)^2$ with

$$\begin{aligned} g^3 &= \frac{1}{\sqrt{2}}[-\sin\theta_1 d\phi_1 + \cos\psi \sin\theta_2 d\phi_2 - \sin\psi d\theta_2] \\ g^4 &= \frac{1}{\sqrt{2}}[d\theta_1 + \sin\psi \sin\theta_2 d\phi_2 + \cos\psi d\theta_2] \\ g^5 &= [d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2]. \end{aligned} \quad (13)$$

This is the S^3 of [1]. We get this, with the following identification of angular coordinates: $\theta_1 \rightarrow \theta_2, \phi_1 \rightarrow \phi_2, \theta \rightarrow \theta_1, \phi \rightarrow \phi_1, \psi \rightarrow \psi$. The RR three form field strength, F_3 , has a quantized flux around S^3

$$\int_{S^3} F_3 = 4\pi^2 N, \quad (14)$$

where the three cycle of S^3 is parameterized by $\theta_2 = -\theta_1, \phi_2 = -\phi_1$ and $\frac{\psi}{2} \rightarrow \psi$, which means that only the range of ψ coordinate is changed and others are same as before ⁴ and the F_3 in this choice of parametrisation becomes

$$F_3 = 2N \sin\theta_1 \sin^2\psi d\theta_1 \wedge d\phi_1 \wedge d\psi. \quad (15)$$

3 Dynamics of p $\overline{D3}$ in MN background

As is being shown in [7, 14] that putting anti-D3 branes in KS backgrounds make these anti-D3 branes to feel a radial force, coming from gravity and F_5 , towards the tip at $r=0$. Hence, the anti-D3 branes will be accumulated at $r=0$.

The same can also be shown to be true for the MN background, where we are probing it with p $\overline{D3}$ branes. The dynamics of p coincident $\overline{D3}$, in the S-dual frame, is described only by the DBI action, (more about it is described in the next subsection) and is

$$S_{DBI} = -T_3 \int STr \left(e^{-\phi} \sqrt{-\det(P[G_{\mu\nu}]) \det(Q_j'^i)} \right). \quad (16)$$

The non-commutator term in this action is

$$-T_3 \int STr \left(e^{-\phi} \sqrt{-\det(P[G_{\mu\nu}])} \right). \quad (17)$$

For the static gauge choice as written below, corresponds to the pull back of $G_{\mu\nu}$ as $e^\phi \eta_{\mu\nu}$ i.e. $P[G_{\mu\nu}] = e^\phi \eta_{\mu\nu}$, follows from eq.(1). Hence the potential that these $\overline{D3}$ are going to see follows from the above action and is

$$V \sim e^\phi. \quad (18)$$

The potential is plotted in figure 1 and it is easy to see that the anti D3 branes will be accumulated at $r=0$.

⁴With this choice of parametrisation the coordinate ψ ranges from $[0, 2\pi]$, but this range of ψ corresponds to the double cover of S^3 . Hence, we shall take the range of ψ to be $[0, \pi]$. (Thanks to O. Aharony for discussion). The volume of a standard unit S^3 is $2\pi^2$. While this parametrisation with the above range of ψ corresponds to a sphere of radius r , with $r = \sqrt{2}$.

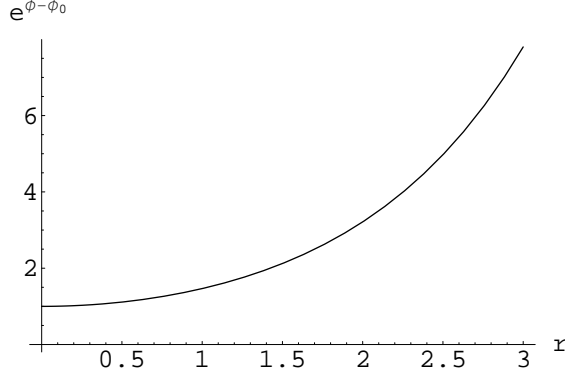


Figure 1: The potential, $V \sim e^{\phi - \phi_0}$ is plotted with r . It follows that the $\overline{D}3$ will be pulled towards $r = 0$.

3.1 Dielectric effects

The dynamics of p coincident anti D3 brane is described by Dirac-Born-Infeld action [12]

$$S_{DBI} = -T_3 \int ST r \left(e^{-\phi} \sqrt{-\det(P[E_{ab} + E_{ai}(Q^{-1} - \delta)^{ij}E_{jb}] + \lambda F_{ab}) \det(Q_j^i)} \right), \quad (19)$$

with ${}^5E_{ab} = G_{ab} + B_{ab}$ and $Q_j^i = \delta_j^i + i\lambda[\Phi^i, \Phi^k]E_{kj}$. $\lambda = 2\pi$, since we are working in $\alpha' = 1$ units and the Chern-Simon action

$$S_{CS} = -\mu_3 \int ST r \left(P \left[e^{i\lambda i_\Phi i_\Phi} (\sum C^{(n)} e^B) \right] e^{\lambda F} \right). \quad (20)$$

The induced metric on the world volume of the anti D3brane, in IR, is assumed to be of the following form:

$$ds_4^2 = \eta^{\mu\nu} dx_\mu dx_\nu, \quad (21)$$

where we have rescaled the coordinates and assumed the static gauge choice i.e. $x^\mu = \sigma^\mu$ and $x^i = \Phi^i = \text{constants}$ ⁶. Hence, there will not be any non vanishing E_{ai} term in the DBI action. The RR two form potential, C_2 is non

⁵The dilaton, ϕ , is not to be confused with one of the angular coordinate that appear in the gauge potential and in the metric of S^2 .

⁶From here onwards the indices μ, ν will denote directions parallel to the brane and i, j for the transverse directions.

zero only along the direction transverse to the world volume of the brane, also we have assumed, that there will not be any F_{ab} term in the action. Hence, the DBI action reduces in IR to ⁷

$$S_{DBI} = -\frac{T_3}{g_s} \int STr \left(\sqrt{-\det(P[(\eta + B)_{\mu\nu}])} \det(Q_j^i) \right). \quad (22)$$

Let us rewrite the above DBI action in the S-dual frame and is given by

$$-\frac{T_3}{g_s} \int STr \left(\sqrt{-\det(P[\eta_{\mu\nu}])} \det(Q_j'^i) \right), \quad (23)$$

with $Q_j'^i = \delta_j^i + i\frac{\lambda}{g_s}[\Phi^i, \Phi^k](G_{kj} + g_s C_{kj})$.

The Chern-Simon action has only one non vanishing term, before taking the $r \rightarrow 0$ limit i.e.

$$S_{CS} = -\mu_3 \int STr \left(P \left[e^{i\lambda i_\Phi i_\Phi C_6} \right] \right), \quad (24)$$

In this limit the complete action of p coincident anti D3 brane in the S-dual frame is described by only eq. (23) in the background of eq.(12), as in the S-dual frame C_6 will be replaced by B_6 in eq.(24). Since, we are considering MN solution described by N D5 branes wrapped on S^2 , which has no B_6 implies that in the S-dual frame there will not be Chern-Simon action. The fields Φ^i 's denote the scalar fields in the direction perpendicular to the brane. If we make these fields to satisfy a non-commutative algebra then the p coincident anti D3 branes will represent a 5 dimensional fuzzy brane, following [12], with world-volume topology $R^4 \times S^2$. For the case in which the number of anti D3 brane is very small in comparison to the quantized F_3 fluxes i.e. $p \ll N$, then Φ remains small relative to the curvature of the spacetime and also with respect to the variations in the three form field strength. In this case we can write, following [7], $C_{ij} = \frac{4\pi}{3} a F_{ijk} \Phi^k$ with $a = \frac{1}{2}$ and the metric in the transverse direction may be assumed locally as $G_{ij} = \delta_{ij}$. The Q_j^i becomes

$$Q_j^i = \delta_j^i + \frac{2\pi i}{g_s} [\Phi^i, \Phi_j] + i \frac{8\pi^2}{3} a F_{kjl} [\Phi^i, \Phi^k] \Phi^l. \quad (25)$$

⁷We have kept B_2 term in the action but eventually it will drop out from eq.(23) for the above stated reason.

Expanding out the square root term in the Lagrangian of eq. (23) and keeping terms to order Φ^4 , we get

$$S_{DBI} = -\frac{T_3}{g_s} \int \left(p - ia \frac{4\pi^2}{3} F_{kjl} \text{Tr} \left([\Phi^k, \Phi^j] \Phi^l \right) - \frac{\pi^2}{g_s^2} \text{Tr} \left([\Phi^i, \Phi^j]^2 \right) \right), \quad (26)$$

which implies that the potential energy is

$$V_p = -L_{DBI} = \frac{T_3}{g_s} \left(p - ai \frac{4\pi^2}{3} F_{kjl} \text{Tr} \left([\Phi^k, \Phi^j] \Phi^l \right) - \frac{\pi^2}{g_s^2} \text{Tr} \left([\Phi^i, \Phi^j]^2 \right) \right). \quad (27)$$

The equation of motion that follows from it for $F_{ijk} = 2f\epsilon_{ijk}$ is

$$[[\Phi^i, \Phi^j], \Phi^j] - 2ig_s^2 af \epsilon_{ijk} [\Phi^j, \Phi^k] = 0. \quad (28)$$

The nontrivial solution to eq.(28) is

$$[\Phi^i, \Phi^j] = -2ig_s^2 af \epsilon_{ijk} \Phi^k. \quad (29)$$

By rescaling the Φ^i 's one can realize it as a $p \times p$ matrix representation of SU(2) algebra

$$[J^i, J^j] = 2i\epsilon_{ijk} J^k, \quad (30)$$

with

$$\Phi^i = -ag_s^2 J^i. \quad (31)$$

The minimum energetic solution would be the p dimensional irreducible representation of SU(2) with the second quadratic Casimir as

$$\text{Tr}[(J^i)^2] = \frac{p}{3}(p^2 - 1). \quad (32)$$

Substituting eq.(31) and using eqs. (32), (30) in eq.(27) gives rise to the potential energy as

$$V_p \simeq \frac{T_3}{g_s} \left(p - \frac{\pi^2}{6} g_s^6 2^4 a^4 f^4 p(p^2 - 1) \right). \quad (33)$$

The value to f follows from the normalization of the three form field strength over S^3 and is given by

$$f = \sqrt{\frac{1}{Ng_s^3}}. \quad (34)$$

Using eq.(34) in eq.(33), we get the potential energy as

$$V_p \simeq \frac{T_3}{g_s} \left(p - \frac{1}{6N^2} \pi^2 p(p^2 - 1) \right). \quad (35)$$

The size of the expanded $\overline{D3}$ branes i.e. fuzzy NS5 brane is

$$R = 2\pi \sqrt{\sum_{i=1}^3 \frac{Tr (\Phi^i)^2}{p}}. \quad (36)$$

Using the equation that Φ 's satisfy namely the $SU(2)$ algebra, we get the radius as

$$R^2 = \frac{\pi^2}{N^2} (p^2 - 1) R_{S^3}^2, \quad (37)$$

with R_{S^3} is the radius of S^3 and is given by $R_{S^3}^2 = N g_s$.

4 The dual NS5-brane point of view

The action of an NS5 brane can be derived by S-dualising the D5 brane action and the dynamics of a D5 brane is governed by the following action [13]

$$\begin{aligned} S_{DBI} &= -T_5 \int d^6 \sigma e^{-\phi} \sqrt{-\det(P[G - B] + 2\pi F)}, \\ S_{CS} &= \mu_5 \int P[C_6]. \end{aligned} \quad (38)$$

In the CS action, we have kept only the non vanishing terms. Going over to S-dual frame, the actions become

$$\begin{aligned} S_{DBI} &= -T_5 \int d^6 e^{-2\phi} \sigma \sqrt{-\det(P[G - e^\phi C_2] + 2e^\phi \pi F)}, \\ S_{CS} &= \mu_5 \int P[B_6] \end{aligned} \quad (39)$$

and this is the action of an NS5 brane. Since, there is no B_6 potential in the background implies that the last term will vanish. Let us recall that the induced metric in the $r \rightarrow 0$ limit is

$$ds_{induced}^2 = dx_\mu dx^\mu + N g_s [d\psi^2 + \sin^2 \psi d\Omega_2^2], \quad (40)$$

where we have rescaled the x_μ coordinates. With this induced metric and putting the NS 5brane in the S^2 of S^3 with radius $\sqrt{N g_s} \sin \psi$, the action of NS 5 brane decomposes, in IR, to

$$S = -\frac{T_5}{g_s^2} \int d^6 \sigma \sqrt{-\det(G_{||}) \det(P[G_\perp - g_s C_2 + 2g_s \pi F])}, \quad (41)$$

with $G_{||}$ denotes the component of the induced metric along the x_μ and G_\perp denotes the component of the induced metric along the S^2 . In IR, $e^\phi \rightarrow e^{\phi_0} \equiv g_s$. The U(1) field strength corresponding to the p anti D3 brane is

$$\begin{aligned} F_{\theta_1 \phi_1} &= \frac{p}{2} \sin \theta_1 \\ 2\pi \int_{S^2} F_2 &= 4\pi^2 p, \end{aligned} \quad (42)$$

the S^2 is characterized by $ds^2 = d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2$ and the 2 form potential, C_2 is

$$\begin{aligned} C_{\theta_1 \phi_1} &= N \sin \theta_1 \left(\psi - \frac{1}{2} \sin 2\psi \right) \\ \int_{S^2} C_2(\psi) &= 4\pi N \left(\psi - \frac{1}{2} \sin 2\psi \right). \end{aligned} \quad (43)$$

Hence, the second determinant in eq.(41) can be rewritten as

$$\int_{S^2} \sqrt{\det(G_\perp + 2\pi g_s F - g_s C_2)} = 4N\pi g_s \left[\sin^4 \psi + \left(\pi \frac{p}{N} - \psi + \frac{1}{2} \sin 2\psi \right)^2 \right]^{\frac{1}{2}} \quad (44)$$

Using eq.(40) and eq.(44) in eq. (41), we get the DBI action as

$$S = \int d^4 \sigma L(\psi), \quad (45)$$

with

$$L(\psi) = -\frac{T_5}{g_s} 4N\pi \left[\sin^4 \psi + \left(\pi \frac{p}{N} - \psi + \frac{1}{2} \sin 2\psi \right)^2 \right]^{\frac{1}{2}} \quad (46)$$

In the case that we are interested in i.e. the static case, the potential energy that follows from the above Lagrangian is

$$V_{eff}(\psi) = -L(\psi). \quad (47)$$

Expanding it for small values of ψ , we get

$$V_{eff}(\psi) \simeq \frac{4\pi^2 T_5 N}{g_s} \left(\frac{p}{N} - \frac{2}{3\pi} \psi^3 + \frac{N}{2\pi^2 p} \psi^4 \right), \quad (48)$$

this has a minimum at $\psi_{min} = \frac{\pi p}{N}$, and the potential energy at this minimum becomes

$$V_{eff}(\psi_{min}) = \frac{4\pi^2 T_5 p}{g_s} \left(1 - \frac{1}{6N^2} \pi^2 p^2 \right). \quad (49)$$

This is the potential seen by p $\overline{D}3$ branes, as calculated in eq.(35). Where we have followed the usual relation between the tension of a D5 brane and with the tension of a D3 brane i.e. $4\pi^2 T_5 = T_3$. The potential i.e. eq.(47) is plotted in figure 2.

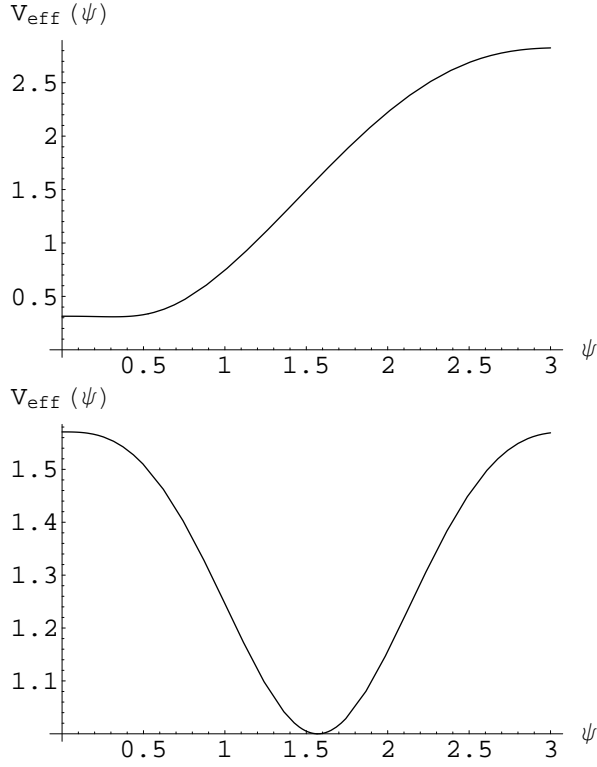


Figure 2: The $V_{eff}(\psi)$ i.e. eq.(47) is plotted with ψ for $\frac{p}{N} = 0.1$ and $\frac{p}{N} = 0.5$ in units of $\alpha' = 1$. It has a stable minima.

5 Fluctuation along the radial direction

Before we start discussing the behavior of the potential energy of N coincident anti D3 branes from the NS5 brane point of view due to fluctuation along the radial direction, let us note the behavior of various fields for small r . The dilaton

$$e^{-2\phi} = e^{-2\phi_0} \left(1 - \frac{8}{9} r^2 + \dots \right). \quad (50)$$

Gauge potential

$$\begin{aligned}
A^1 &= -(1 - \frac{2}{3}r^2 + \dots)d\theta, \\
A^2 &= (1 - \frac{2}{3}r^2 + \dots)\sin\theta d\phi, \\
A^3 &= -\cos\theta d\phi.
\end{aligned} \tag{51}$$

The geometry in this case with $\theta_1 = -\theta_2$, $\phi_1 = -\phi_2$, and $\psi \rightarrow 2\psi$ choice, to quadratic in r , becomes

$$\begin{aligned}
ds^2 \approx e^{\phi_0} &\left(dx^\mu dx_\mu + N(d\psi^2 + \sin^2\psi d\Omega_2^2) + \right. \\
&\left. r^2[\frac{4}{9}dx^\mu dx_\mu + N d\Omega_2^2 + N\frac{dr^2}{r^2} + \frac{4}{9}N d\psi^2 - \frac{2}{9}N \sin^2\psi d\Omega_2^2] \right), \tag{52}
\end{aligned}$$

with the expansion of functions $a(r)$ and $h(r)$ as

$$\begin{aligned}
a(r) &= 1 - \frac{2}{3}r^2 + \dots, \\
e^{2h} &= r^2(1 - \frac{4}{9}r^2 + \dots).
\end{aligned} \tag{53}$$

The 3 form RR field strength i.e. eq.(5) with the above choice of angular coordinates θ_i and ϕ_i becomes

$$\begin{aligned}
F_3 = N &\left((2\sin\theta_1 \sin^2\psi - \frac{4}{3}r^2 \sin\theta_1 \sin^2\psi + \frac{2}{3}r^2 \sin\theta_1) d\theta_1 \wedge d\phi_1 \wedge d\psi + \right. \\
&\left. \frac{2r}{3} \sin 2\psi \sin\theta_1 dr \wedge d\theta_1 \wedge d\phi_1 \right)
\end{aligned} \tag{54}$$

It is easy to see that the 2 form potential takes the following form

$$C_2 = N \left([(1 - \frac{2}{3}r^2) \sin\theta_1 (\psi - \frac{1}{2} \sin 2\psi) + \frac{2}{3}r^2 \psi \sin\theta_1] d\theta_1 \wedge d\phi_1 \right). \tag{55}$$

The dynamics of an NS5 brane is described by

$$\begin{aligned}
S &= -T_5 \int e^\phi \sqrt{-\det(P[e^{-\phi}G - C_2] + 2\pi F)} \\
&= -T_5 \int e^{-2\phi} \sqrt{-\det(P[G - e^\phi C_2 + 2\pi e^\phi F])} \\
&\approx -\frac{T_5}{g_s^2} \int (1 - \frac{8}{9}r^2) \sqrt{-\det(P[G - e^\phi C_2 + 2\pi e^\phi F])}.
\end{aligned} \tag{56}$$

In the 1st line we have S-dualised the D5 brane action and in the last line the dilaton is expanded and kept to quadratic in r and as before the Chern-Simon action in the S-dual frame vanishes as there is no B_6 potential to which it can couple, since our background is that of D5brane MN solution [2].

We can rewrite $G - e^\phi C_2 + 2\pi e^\phi F$, using eq.(50) as

$$G - g_s(1 + \frac{4}{9}r^2)C_2 + 2\pi g_s(1 + \frac{4}{9}r^2)F, \quad (57)$$

with identifying $g_s = e^{\phi_0}$. Hence, the determinant of eq.(56) can be rewritten, using eq.(52), as

$$\det(G_{||})\det(G_{\perp} - g_s(1 + \frac{4}{9}r^2)C_2 + 2\pi g_s(1 + \frac{4}{9}r^2)F), \quad (58)$$

where $G_{||}$ and G_{\perp} denotes components of metric along x^μ and along the Ω_2 directions, respectively. Explicitly, G_{\perp} is given by

$$G_{\perp} = Ng_s \sin^2 \psi (1 - \frac{2}{9}r^2) d\Omega_2^2 + Nr^2 g_s d\Omega_2^2. \quad (59)$$

Evaluating the second determinant of eq.(58), we get

$$N^2 g_s^2 \sin^2 \theta_1 \left(\left[\left(1 - \frac{2}{9}r^2\right) \sin^2 \psi + r^2 \right]^2 + \left[- \left(1 + \frac{4}{9}r^2\right) \left(\psi - \frac{1}{2} \sin 2\psi + \frac{r^2}{3} \sin 2\psi\right) + \frac{\pi p}{N} \left(1 + \frac{4}{9}r^2\right) \right]^2 \right). \quad (60)$$

The number p that appears in the above expression comes through the 2 form field strength, F_2 , which corresponds to the number of anti D3 branes that is being used to probe the D5 brane background of Maldacena-Nunez, as studied in section 3. Now integrating it over the two sphere, S^2 , and keeping terms to quadratic in r , gives

$$\begin{aligned} & \int_{S^2} \sqrt{\det(G_{\perp} - g_s(1 + \frac{4}{9}r^2)C_2 + 2\pi g_s(1 + \frac{4}{9}r^2)F)} \\ &= 4\pi N g_s \left(\sqrt{\sin^4 \psi + \left(\pi \frac{p}{N} - \psi + \frac{1}{2} \sin 2\psi\right)^2} + \frac{\left[2\sin^2 \psi - \frac{4}{9}\sin^4 \psi + 2\left(\frac{4p}{9N}\pi - \frac{4}{9}\psi - \frac{1}{9}\sin 2\psi\right)\left(\pi \frac{p}{N} - \psi + \frac{1}{2}\sin 2\psi\right) \right] r^2}{2\sqrt{\sin^4 \psi + \left(\pi \frac{p}{N} - \psi + \frac{1}{2}\sin 2\psi\right)^2}} \right). \end{aligned} \quad (61)$$

Hence the potential that follows from eq.(56), with the induced metric on the remaining four directions as $ds_4^2 \approx (1 + \frac{4}{9}r^2)\eta_{\mu\nu}dx^\mu dx^\nu$, is

$$V_{eff}(\psi, r) \approx \frac{4\pi NT_5}{g_s} \times \left(\sqrt{\sin^4\psi + (\pi\frac{p}{N} - \psi + \frac{1}{2}\sin 2\psi)^2} + \frac{\left[\sin^2\psi - \frac{2}{9}\sin^4\psi + (\frac{4p}{9N}\pi - \frac{4}{9}\psi - \frac{1}{9}\sin 2\psi)(\pi\frac{p}{N} - \psi + \frac{1}{2}\sin 2\psi) \right] r^2}{\sqrt{\sin^4\psi + (\pi\frac{p}{N} - \psi + \frac{1}{2}\sin 2\psi)^2}} \right). \quad (62)$$

The potential is plotted in figure 3, for $p/N=0.1$ and for $p/N=0.5$. It is easy to note that eq.(47) follows in the $r \rightarrow 0$ limit. The coefficient of r^2 in the second term of eq.(62) is always positive, and is plotted in figure 3 for $p/N=0.1$, which means that the system will always be driven towards the $r=0$.

6 Conclusion

Probing MN solution of N D5 branes with a stack of p anti D3 branes in IR reveals that the potential energy seen by these probe branes is stable. Which means that this probe brane have broken all the supersymmetry of MN solution⁸. We have also tried to understand some kind of geometric transitions between MN backgrounds with anti D3 branes and the fuzzy NS5 branes. However, it would be interesting to understand it by including both the Dp and anti Dp branes and studying it not in the S-dual frame.

The analysis is done as stated above and in the introduction that we have considered the probe action of p coincident anti D3 branes and going over to the S-dual frame we have the following terms in the DBI action: metric, two form potential, C_2 coming from the three form field strength, $F_3 = dC_2$, and there is no F_2 , the U(1) field strength. Note that we are keeping the term B_2 in the DBI action before going to the S-dual frame even though we are considering the D5 brane solution of MN, where there is no B_2 potential but eventually C_2 drops out in the S-dual frame because it has no nonzero component along the x_μ directions. The Chern-Simon term drops out in the S-dual frame as we do not have a B_6 term in the background. Hence, we left with only the DBI action. Taking the $r \rightarrow 0$ limit, we analyzed this action in the MN background and found that the p anti D3 brane can become a

⁸Thanks to Ofer Aharony for the discussion.

fuzzy NS5 brane, which minimizes the energy. Now, studying the system from the NS5 brane point of view, with the charge corresponding to that of the p anti D3 branes on the world volume of the NS5 brane, we found that the potential energy matches with that found in the nonabelian analysis. Note, we construct the NS5 brane action by starting with the action of a D5 brane which contains metric, B_2 and F_2 in the DBI action and a six form potential, C_6 , in the CS action. Going over to the S-dual frame, we left with only the DBI action as we do not have any B_6 term in the background, which can couple to NS5 brane. Hence, in this case also we left with only the DBI action. As mentioned above, the leading term in the potential matches with that of nonabelian potential of p anti D3 brane and the full potential of NS5 brane gives us a minimum which is stable. Repeating the same NS5 brane analysis but a little bit away from the $r=0$ and keeping terms to quadratic in r in potential also makes us to draw the same conclusion as the potential is stable at $r=0$.

It seems from the analysis we did in section 3 and in 4 that if we replace anti D3 branes by D3 branes then the result do not changes since the CS action drops out of the probe brane analysis, but we believe that doing the analysis not in the S-dual frame might resolve this apparent ambiguities.

Let us note few things if we are not doing the analysis in S-dual frame. First the B_2 term will drop out from the DBI action of eq.(38) as we are in the background of D5 branes of MN but there will be a CS action containing $i_\Phi i_\Phi C_6$. Hence, the difference will be that of including the CS action and simultaneously loosing a term from DBI action. Whereas in the S-dual frame, we have no CS action but we have a C_2 term in eq.(39). So, the over all square root term in the potential will be changed to a term containing square root and the one coming from CS action.

It would also be interesting to study the probe brane behavior in IR both for the MN as well as the KS backgrounds but not going to S-dual frame for both Dp and \overline{Dp} branes. Note that the dilaton dependence of a bunch of NS5 (D5) branes and NS5 (D5) branes wrapped on a two sphere behave in the same way in IR. Hence, it would be interesting also to study these two and to see the similarities and differences, if any.

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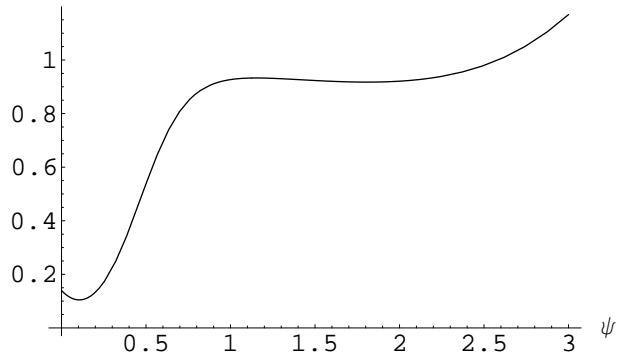
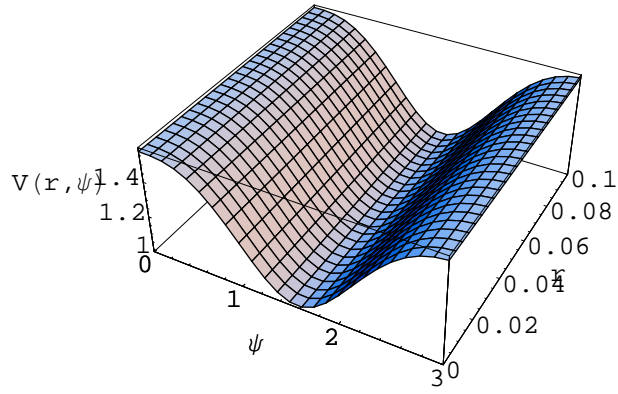
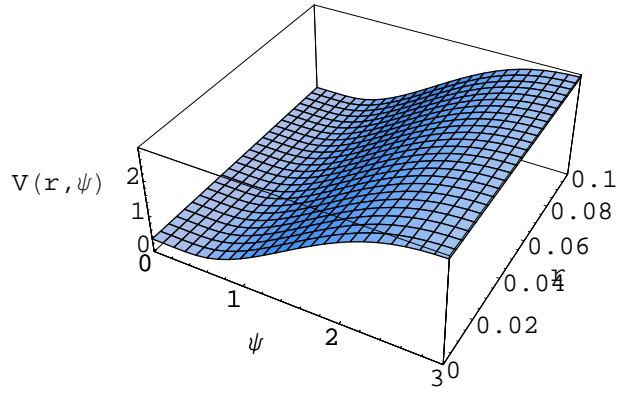


Figure 3: The $V_{eff}(\psi, r)$ i.e. eq.(62) is plotted with ψ and r for $\frac{p}{N} = 0.1$ and $\frac{p}{N} = 0.5$ respectively in units of $\alpha' = 1$ and the last curve is the coefficient of r^2 of eq.(62) for $p/N=0.1$. 18